RELIABILITY OF TREND LINES OBTAINED FROM VARIABLE COUNTS

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Abstract: **Standard errors (SE) of trend lines arising from highly variable counts of animals are derived analytically given the variability of the counts, the number of years of trend monitoring, the number of replicate counts each year, and certain assumptions. Examples of the analytical solution's use and graphs of expected trend line variability are presented. When counts are highly variable, performing multiple counts each year is shown to be the only way to achieve precision of a population trend estimate within a short (<12 years) time frame. Computer simulations to test the robustness of the analytical treatment under assumption violations show that it never overestimates and sometimes underestimates the true variability of trend lines. Accordingly, when count variability cannot be reduced, I suggest increasing the number of replicate counts each year above that indicated by the analytical solution to ensure that confidence limits (CL) about the trend line include the population's true rate of growth.**

J. WILDL. MANAGE. ***50(1):165–171***

A common difficulty facing wildlife managers is estimating population size with acceptable precision. Population estimation is especially difficult for rare and/or secretive species whose status may nevertheless be of concern. Counts of these species often are imprecise, and confidence intervals on resulting estimates are correspondingly broad. Eberhardt (1978) treats this topic in detail, suggesting procedures for determining adequate sample sizes and giving rough ideas of the variability to be expected from indirect indices as well as counts.

Where precise counts are difficult or impossible, attempts are often made to monitor general trends over time. A case in point is the current effort to monitor trends of remnant populations of grizzly bears (*Ursus arctos*) in the contiguous United States. Population monitoring is called for in the grizzly bear recovery plan (U.S. Fish and Wildl. Serv., unpubl. rep., Denver, Colo., 1982), but grizzly populations are notoriously difficult to census with reasonable precision (Erickson and Siniff 1963). Little attention has been paid to the precision required of counts used for year-to-year trend monitoring. The implicit assumption often is that data lacking the quality and quantity needed for population estimation can at least be used for trend monitoring. However, observed trends themselves may be misleading if based on data that are highly variable.

In this paper I use an analytical procedure to calculate the variability of observed trend lines. This procedure requires assumptions that may not be met in field situations. Accordingly, I also present simulation modeling experiments that test the robustness of the calculation when these assumptions are violated. (I define a trend line as the slope of the least squares regression of the logarithm of observed numbers over time [r,] and quantify trend line reliability by the SE of the slope [SE,]).

An earlier version of this paper appeared in a report to the U.S. Fish and Wildl. Serv. on the feasibility of a trend monitoring program for grizzly bears in their remaining range in the lower 48 states. I thank R. M. Cormack for suggesting the analytical solution. L. Metzgar reviewed the manuscript and added greatly in its development. R. Hollister provided statistical assistance. Improvements also were suggested by R. Klaver, I. J. Ball, and W. R. Clark.

Financial support came from the U.S. Fish and Wildl. Serv., through the office of C. Servheen, Grizzly Bear Recovery Coord.

DIRECT CALCULATION OF TREND LINE ERROR: ANALYTICAL TREATMENT

Consider the following scenario-we wish to obtain the trend of a wildlife population that is changing at a constant rate, such that taking the natural logarithm (ln) of the number in the population each year would result in the familiar equation producing a straight line:

$$\ln N_t = \ln N_o + rt, \qquad (1)$$

where N = number, t = time, and r = yearly instantaneous/capita rate of change. When a population is subjected each year to trend counts (e.g., from aircraft) that are both incomplete and variable, N will be unknown. If trend counts are substituted for N, the resulting slope can be considered r_o, the observed rate of change. Given the following assumptions: (1) population change is exponential (i.e., the slope of r is linear); (2) counts are independent; (3) counts are lognormally distributed (they may take continuous, positive values or they may exceed the actual population); and (4) the mean percentage of the population counted (mean sightability) is constant at all population levels, the value r_o will be normally distributed with an expected value r and variance:

$$\sigma^2/\Sigma(t_i - \bar{t})^2, \qquad (2)$$

where σ^2 = the common variance of the logarithm of the counts about the trend line. Taking n counts each year over k years results in:

$$SE(r_o) = \sqrt{12\sigma^2/nk(k^2 - 1)}. \qquad (3)$$

Generating σ^2 may cause problems. Usually, field biologists do not have this information at the beginning of a study. However, Eberhardt (1978) gave estimates of coefficients of variation (CV = $\sigma/\bar{x}$) to be expected from counts of terrestrial animals and noted that they generally were constant within taxa over a broad range of population densities. CV's for counts of terrestrial mammals seem to vary from 30 to >100%. Goddard's (1967) 18 replicate counts of black rhinoceros (*Diceros bicornis*) have a CV of 42%. A CV ≥50% is not uncommon with counts of rare species, such as grizzly bears; e.g., Reynolds (1974). Assuming lognormality, the σ^2 required for equation (3) is:

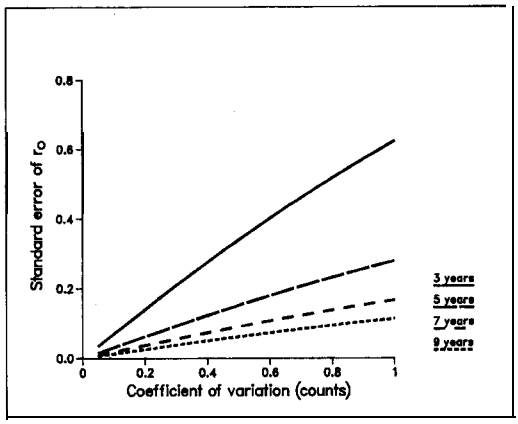

Fig. 1. The standard error of the observed population trend (SE[r_o]) as a function of count coefficient of variation (CV) for 3, 5, 7, and 9 years of trend monitoring. A single count each year is assumed. Calculation is from equation (3).

$$\sigma^2 = \ln(1 + CV^2), \qquad (4)$$

where CV = the estimated coefficient of variation of counts. If within year replicate counts are performed, they may be used to refine the estimate of CV as the study progresses. With a large enough sample, the CV based on the results of the experiment will be preferred over that guessed a priori. However, if counts are taken only once yearly, distinguishing count variability from true population change may not be possible.

Thus, given the previous assumptions, knowledge of the probable CV for trend counts leads directly to estimation of the error associated with r_o and potential quantification of the observed trend line's reliability. The relationships between count CV, number of within year replicate counts, number of years of population monitoring, and the resultant SE(r_o) are illustrated in Figures 1, 2, and 3. Note that SE(r_o) increases with CV less as trend is measured over a longer time period (Fig. 1). A similar pattern is seen in Figure 2, where the advantage of taking replicate counts each year is shown. For a given CV of counts, the decrease in SE(r_o) obtained by increasing the length of time the trend is monitored is not linear (Fig. 3). For example, increasing the number of years of trend monitoring from 3 to 6 produces a greater reduction in the variability of r_o than does a similar increase from 6 to 9 years.

For planning purposes, one might want to know combinations of count CV, years of trend

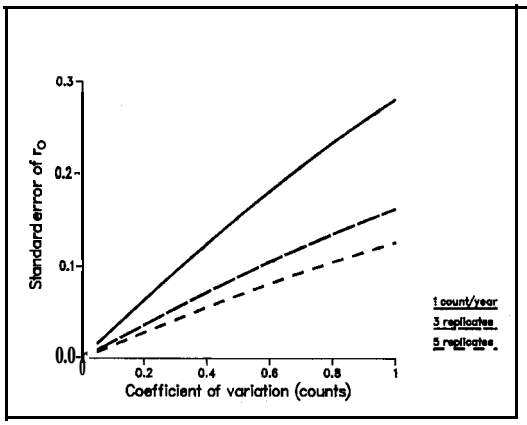

Fig. 2. The standard error of the observed population trend ($SE[r_o]$) as a function of count coefficient of variation (CV) for 1, 3, and 5 replicate counts each year. All calculations are for a B-year monitoring period. Calculation is from equation (3).

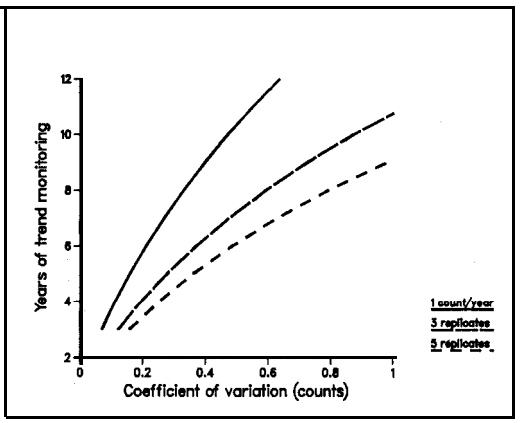

Fig. 4. Curves denoting combinations of trend-monitoring time period, count coefficient of variation (CV), and numbers of within year replicates that result in standard error of observed population trends ($SE[r_o]$) of 0.05. For each curve, the area above and to the left produces $SE(r_o)$ ~0.05; the area below and to the right produces higher $SE(r_o)$ values.

monitoring, and number of within year replicates that result in a prespecified precision of the observed trend. Such a plot is presented in Fig. 4, where $SE(r_o)$ values of 0.05 are denoted by curves for 1, 3, and 5 counts each year. The following 2 hypothetical examples illustrate the direct calculation of the precision of population trend estimates from variable counts.

Example 1: The Precision of a Trend Estimate

An expanding ungulate herd has been monitored for the past 9 years by counts from yearly aerial flights. The biologists involved estimate

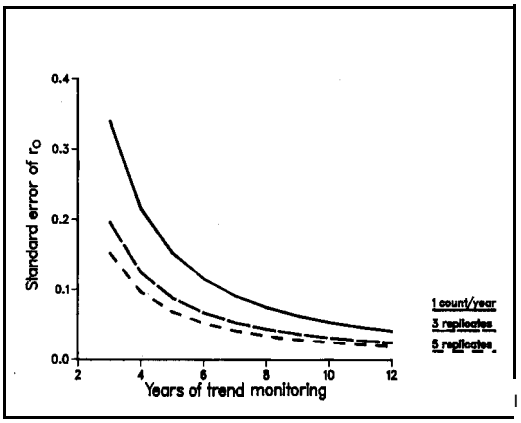

Fig. 3. The standard error of the observed population trend ($SE[r_o]$) as a function of trend monitoring time period for 1, 3. and 5 replicate counts each year. All calculations are for a count coefficient of variation (CV) of 0.50. Calculation is from equation (3).

that they observe, on average, half the animals present. Based on previous studies and their best judgement, they estimate the underlying CV of counts to be about 25%. By regressing the ln of each year's count against time, they have estimated the herd's instantaneous rate of increase (r_o) to be 0.05, corresponding to a finite rate of increase of approximately 5.1%/year. They now wish to calculate the precision of their estimate of yearly population growth.

Assuming a CV of 0.25, the appropriate σ^2 term from equation (4) is 0.061. Substituting this into equation (3), where n = 1 (because a single count was taken each year) and k = 9 years of trend monitoring, the SE of their estimate of r is 0.032. Because r_o is normally distributed, the probability is approximately 0.68 that the actual value of r lies between 0.018 and 0.082 (0.05 ± 0.032 [SE]), corresponding to finite rates of increase between 1.8 and 8.5% yearly.

Example 2: Planning a Trend Monitoring Effort

An agency wishes to monitor the trend of a small bear population from yearly counts. Previous experience has suggested that counts of bears in this area are highly variable; even under the best circumstances, a CV of 50% can be expected. Further, the agency feels it necessary to determine the population's trend over a 5-year period and would like its estimate of r to be precise to within 0.05 at the 68% con-

Table 1. Summary of the slope of the observed population trend line, r_o, and its standard error, $SE(r_o)$, as calculated by 3,000 simulations that violated assumptions of equation (3), compared with values obtained from the direct solution (eq. 3).

Assumptions violated	Simulated values compared to equation (3)	
	r_o	$SE(r_o)$
None	**No change**	**No change**
Exponential growth	**No change**	**No change**
Independence of counts	**No change**	**Higher**
Counts log-normally distributed	**No change**	**Higher**
Constant mean sightability		
Techniques improve with time	**Biased higher**	**No change**
Observability varies positively with density	**Greater in absolute value**	**No change**
Observability varies negatively with density	**Lesser in absolute value**	**No change**

fidence level. Can this precision be achieved, and if so, at what cost?

Referring to Figure 4, the location corresponding to a CV of 0.50 over 5 years is below the 5 replicates/year contour. Equation (3) can be rearranged to solve for n, the number of required within year replicate counts:

$$n = 12\sigma^2/k(k^2 - 1)(SE[r_o])^2, \qquad (5)$$

where all variables are as in equation (3), and $SE(r_o)$ is the ***desired*** level of precision. In this case, with a CV of 0.50 (σ^2 value of 0.223), a desired $SE(r_o)$ of 0.05, and $k = 5$, ≥ 9 independent replicate counts/year are required. If replicate flights are considered too expensive, the alternatives are lengthening the trend-monitoring period or reducing the desired precision of the estimate of trend.

Note that precision of r to ≤ 0.05 at the 68% confidence level reduces but does not eliminate the probability of wrongly deciding the population's gross trend; i.e., up or down. Suppose the bear population is decreasing at 5%/year. With this intensity of sampling, there remains a 16% probability (i.e., 0.5[100–68]%) that counts will indicate a stationary or increasing population over the 5 years.

SIMULATIONS TO EXAMINE THE IMPORTANCE OF ASSUMPTIONS

Given the previous assumptions, the variability in observed trend lines can be calculated directly as in the 2 examples. However, biologists may not be comfortable with many of the assumptions. Managed populations rarely grow exponentially, counts may not be truly independent of each other, sightability may vary, and the underlying distribution of counts may be unknown. Therefore, it is instructive to examine the validity of equation (3) under a variety of assumption violations that might occur in the field.

I developed stochastic computer models that simulated variable counts from hypothetical big game populations. All simulations contained an identical core routine in which counts were randomly obtained from a population of 100 "animals" according to a chosen frequency distribution with a prespecified mean and variance. Up to 5 replicate counts were made each model year, and runs included trend-monitoring periods of 3-12 years. Simulated CV's of counts varied from 5 to 100%. Most simulations assumed a mean sightability of 0.50, a typical proportion of large mammals observed during

Table 2. The standard error of population trend lines, $SE(r_o)$, comparing the direct method (eq. 3) and 3,000 simulations which did not violate 4 a priori statistical assumptions.

Within year replicates	Years of trend-monitoring	cv of counts	$SE(r_o)$ Direct	$SE(r_o)$ Simulated
1	5	0.15	0.0472	0.0468
		0.75	0.2113	0.2137
	9	0.15	**0.0193**	**0.0188**
		0.75	0.0862	0.0884
3	5	0.15	0.0272	0.0281
		0.75	0.1220	0.1268
	9	0.15	**0.0111**	**0.0108**
		0.75	**0.0498**	**0.0497**

Table 3. The standard error of population trend lines, $SE(r_o)$, comparing the direct method (eq. 3) and 3,000 simulations in which the total variance of counts was partitioned into within and between years segments. Data are from runs in which the between year variance was 2 × the within year variance. Trend line periods are 5 years.

Within year replicates	cv of counts	$SE(r_o)$ Direct	$SE(r_o)$ Simulated
3	0.15	0.0272	**0.0291**
	0.30	0.0536	0.0572
	0.45	0.0784	0.0873
5	0.15	0.0211	**0.0294**
	0.30	0.0415	0.0576
	0.45	0.0607	0.0881

Table 4. The standard error of population trend lines, $SE(r_o)$, comparing the direct method (eq. 3) and 3,000 simulations in which counts were distributed normally. A single count/year is assumed throughout.

Years of trend monitoring	cv of counts	$SE(r_o)$ Direct (lognormal)	$SE(r_o)$ Simulated (normal)
5	0.15	0.0472	0.0492
	0.35	0.1075	0.1393
7	0.15	0.0282	0.0288
	0.35	0.0642	0.0870
9	0.15	0.0193	0.0200
	0.35	0.0439	0.0605

aerial surveys; e.g., Erickson and Siniff (1965), Caughley (1977), Floyd et al. (1979). However, when simulating large CV's of counts (>50%), mean sightabilities of 0.10 were used to avoid substantial truncation of log-normal distributions. Exponential growth was modeled in the initial simulations by applying the constant rate $e^r = \lambda$ to the population at the end of each simulation year, where r took values from -0.2 to 0.2 and included zero. After the completion of each modeled trend-monitoring period, the ln's of the counts were regressed against time, yielding r_o. For each parameter combination, the entire process was repeated 3,000 times, and the mean and SE of the 3,000 r_o values calculated. Results from the simulations are summarized and compared with those from the direct solution in Table 1.

The initial simulations met all the necessary assumptions for the direct solution except that counts were not distributed strictly lognormally; because only whole animals were assumed to be counted, counts took only integer values. Variability of observed trend lines over all simulated parameter combinations essentially was identical to that calculated by equation (3) (Table 2). Having thus validated the general model, the previous assumptions were systematically altered and changes in r_o and $SE(r_o)$ examined.

Assumption 1. Exponential Growth of Monitored Population

I deliberately violated the assumption of exponential growth by simulating growth in which a constant proportion of the initial population was added (or subtracted) each year; e.g., population sizes of 100, 95, 90, 85, . . . etc., in years 1, 2, 3, 4, . . . etc. Under this scenario, r is not strictly appropriate as a predictor of future population growth, but r_o satisfactorily approximated the expected r prevailing over simulated trend-monitoring periods. Similarly, there were no discernible differences in the variability of r_o between simulations with additive growth and that expected under equation (3). Thus the direct solution appeared robust to this kind of violation in the assumption of exponential growth.

Assumption 2: Independence of Counts

The assumption of independence of counts can be violated in several ways. I considered it most likely that counts closest in time are most similar to each other, either because replicate counts are conducted temporally so close to each other that the animals are in approximately the same locations, or because sightability varies less within each year than between years. Examples of the latter scenario are counts where the amount of yearly snow cover affects the number of animals seen or where prolonged summer drought forces animals into areas of dense vegetation where they cannot be seen. In either case counts within each year may be considered as coming from a distribution with a smaller variance than the overall variance. This was simulated by subdividing the total count variance into within and between year segments. The SE's of trend lines obtained from these simulations were greater than in the direct solution (Table 3). The discrepancies were greatest when between year variances were larger than within year variances, and when 5 replicate counts were made rather than 3. In effect, taking more within year counts simply allowed full expression of between year differences in sightability.

Assumption 3: Log-normal Distribution of Counts

Counts of large animals can be expected to be log-normally distributed if independent factors affect the number counted in a multiplicative way (Pielou 1975, Eberhardt 1978). However, if independent factors influence counts in an additive way, a normal distribution of counts can be expected. Studies reporting replicate counts of black rhinoceros (Goddard 1967) and grizzly bears (Erickson and Siniff 1965) do not resolve the issue; both data sets are consistent with either normal or log-normal distributions. Therefore, I simulated the $SE(r_o)$ under the alternative assumption of normally distributed counts. When counts are log-normally

the logarithm of counts are, of mally distributed. But when counts lly distributed, their logarithms are ıd their variance greater than σ^2, even 2 untransformed distributions have variances. Not surprisingly, simulated :s obtained from normally distributed ıd greater variability than those ob- rom counts distributed lognormally

ɔtion 4: Constant Mean ɔility

.tions of assumptions I-3 did not affect ɔected slope of the trend line, r_o, only However, when mean sightability var- .tematically, the expected observed trend was biased. The response of trend lines to y scenarios of systematic variation in mean .bility was not surprising. First, when the y of field personnel to locate and count .als improves with experience, the expect- ɔunt increases with time. Therefore, a sta- population may appear to increase simply ıuse counting techniques have improved. simulated r_o values paralleled the simulated ceases in mean sightability through time. :ond, when the behavior or habitat use of the imals in question varies with their density, ;htability is a function of density. For exam- e, if animals at low densities occupy preferred ıbitats in which they are seen easily, whereas ıcreasing density forces surplus animals into uboptimal habitats where sightability is lower, :xpected counts will be a decreasing function ɔf density. Conversely, if optimal habitat is thickly vegetated, and increasing density forces some animals into areas where they are observed more easily, expected counts will in-' crease with density. When mean sightability is a decreasing function of density, r_o underestimated the magnitude of r in both directions. The observed trend overestimated the magnitude of population change when mean sightability was modeled as an increasing function of density.

MANAGEMENT IMPLICATIONS

Simulations showed that, under conditions typically confronting the field biologist, direct calculation by equation (3) never overestimates $SE(r_o)$ and may underestimate it. If the estimated CV used to calculate $SE(r_o)$ is in reality composite of between and within year variability (exclusive of the variability caused by actual population change), the reduction in $SE(r_o)$ achieved by taking multiple counts each year may be less than indicated by equation (3). Additionally, if the underlying distribution of counts is normal rather than lognormal, the calculated $SE(r_o)$ will be underestimated.

Notwithstanding these slight discrepancies, the direct calculation appears generally robust and is useful in a variety of situations. It can generate confidence limits on a point estimate of population growth (as in Example 1) or suggest sample sizes necessary to achieve specified goals of precision (as in Example 2). Given that usually not much is known about the distribution or variance of counts, I suggest increasing the length of monitoring periods or the number of replicate counts taken each year over that indicated by equation (3) to ensure that the variability of the resultant r_o is not underestimated.

Some big game biologists may feel uncomfortable with the use of r, preferring instead the finite rate of growth λ, from which the calculation of percent change/year is simply $(\lambda - 1) \times 100\%$. However, r is more appropriate when the precision of the statistic is of interest, because its variance is independent of its magnitude; i.e., the calculated SE is applicable at any value of r. Hence, equation (3) is equally valid for populations that are changing rapidly in either direction, as well as those that are stationary. Note, **however,** that when back transformed to finite rates, the variability in the resultant λ is dependent on the value of λ. For example, when $r_o = 0.20$ and $SE(r_o) = 0.05$, the transformed 68% CL's around $e^r = \lambda$ are 1.162-1.284, giving increases of between 16 and 28%/year (range = 12%). When $r = -0.20$, the identical $SE(r_o)$ results in declines of 14-22%/year (range = 8%). In the latter case, the precision of the back-transformed estimate is greater although the variability about the untransformed estimate is the same in both cases.

Managers using trend information obtained from variable counts should know something about the accuracy and precision of their trend estimate. This investigation has shown that trend estimates based on highly variable counts require multiple counts for high precision, by either taking replicate counts each year or calculating trend only after a sufficiently long period of time.

Proper survey design and strict standardiza-

tion of procedures can reduce count variability but seldom sufficiently to eliminate the concerns raised here. Investigators can lengthen the monitoring period but not without cost. By insisting on waiting until enough years have passed that the apparent trend is considered reliable, smaller fluctuations in population size may be obscured. Additionally, as the time period increases, the consequences of erring in interpreting trends become greater. In Example 1, the 68% CL's around the annual estimate of 5.1% yearly growth ($r = 0.05$), as calculated after 9 years of monitoring, were 1.8 and 8.5%. In year 9, these limits implied population sizes of from 115 to 193% of the population in year 0, calculated as $\exp(r_o \pm [SE(r_o)(t - 1)])$. If the same $SE(r_o)$ could have been achieved within 5 years, equivalent limits around the estimated population size in year 5 would have been 107–139% of the original population, a much smaller range. When counts are highly variable and time-frame limited, taking replicate counts each year is the only method that can return reliable trend information. The time between such replicate counts should be short enough to minimize changes in the size of the sampled population but long enough to ensure the independence of each count.

Finally, it must be remembered that precision of a trend estimator is of little value if the estimate itself is biased. Changing sightability, either with time or population density, biases r_o, so calculated CL's may not include the population's actual rate of growth. It behooves managers using trend data from partial counts to determine if the animals seen during trend counts are a constant fraction of the target population. If sightability is constant the information presented here provides a method for determining the precision of the estimate of population trend. If sightability varies the relationships between observer experience, population size, and resultant sightability should be elucidated before trend lines can have much meaning.

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Received *10 July 1985.*
Accepted 12 August 1985.